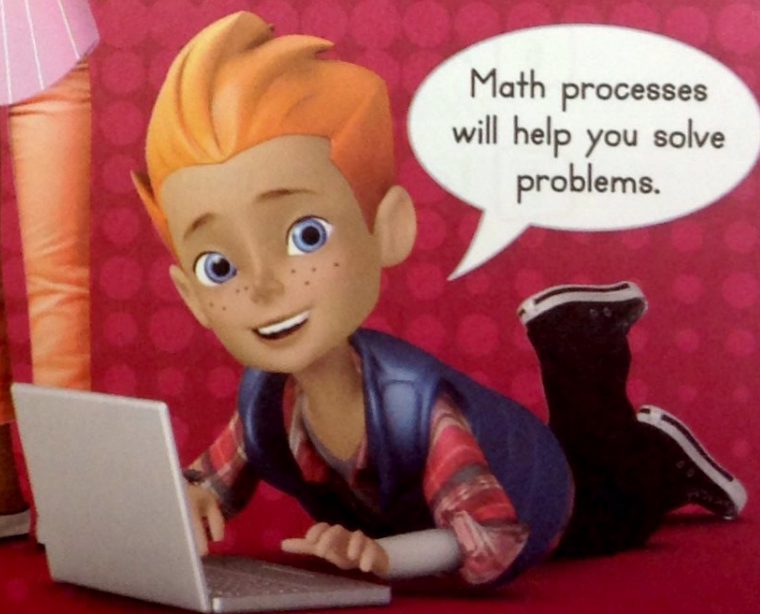


# Problem Solving Handbook



Math processes are ways we think about and do math.



Math processes will help you solve problems.

# Math Processes

- 1 Make sense of problems and persevere in solving them.
- 2 Reason abstractly and quantitatively.
- 3 Construct viable arguments and critique the reasoning of others.
- 4 Model with mathematics.
- 5 Use appropriate tools strategically.
- 6 Attend to precision.
- 7 Look for and make use of structure.
- 8 Look for and express regularity in repeated reasoning.

There are good Thinking Habits for each of these math processes.





# Make sense of problems and persevere in solving them.

Good math thinkers make sense of problems and think of ways to solve them.

If they get stuck, they don't give up.



Anton buys 2 laptops for \$600 each and a printer that costs \$99. He has a \$50 off coupon. How much does Anton pay in all?

Here I listed what I know and what I am trying to find.

What I know:

- Anton has a \$50 coupon.
- Anton buys 2 laptops for \$600 each.
- Anton buys a printer for \$99.

What I need to find:

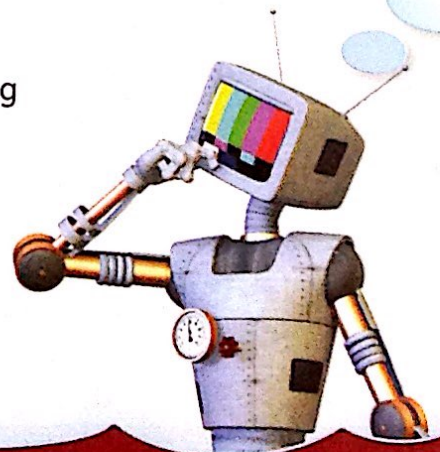
- The total amount Anton will pay.



## Thinking Habits

Be a good thinker! These questions can help you.

- What do I need to find?
- What do I know?
- What's my plan for solving the problem?
- What else can I try if I get stuck?
- How can I check that my solution makes sense?





2

# Reason abstractly and quantitatively.

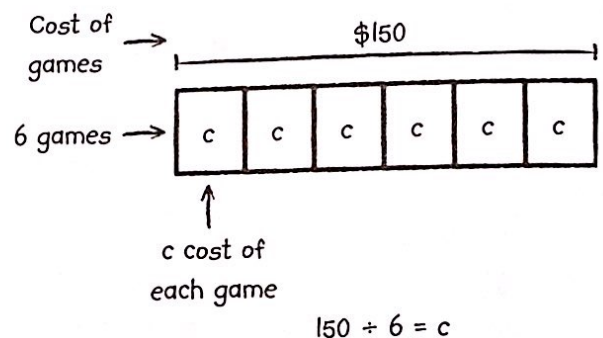
Good math thinkers know how to think about words and numbers to solve problems.



I drew a bar diagram that shows how the numbers in the problem are related.



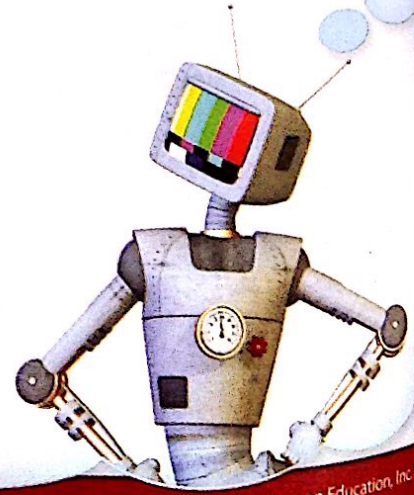
Derrick buys 6 games that cost a total of \$150. How much does each game cost?



## Thinking Habits

Be a good thinker! These questions can help you.

- What do the numbers and symbols in the problem mean?
- How are the numbers or quantities related?
- How can I represent a word problem using pictures, numbers, or equations?





3

## Construct viable arguments and critique the reasoning of others.

Good math thinkers use math to explain why they are right. They can talk about the math that others do, too.



Molly says that every fraction whose denominator is twice as great as its numerator is equivalent to  $\frac{1}{2}$ . Do you agree? Explain.

I wrote a clear argument with words, numbers, and symbols.

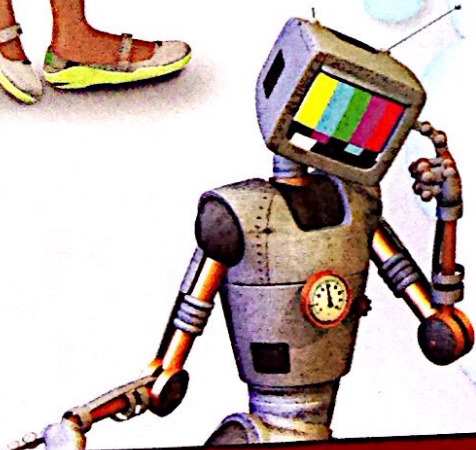
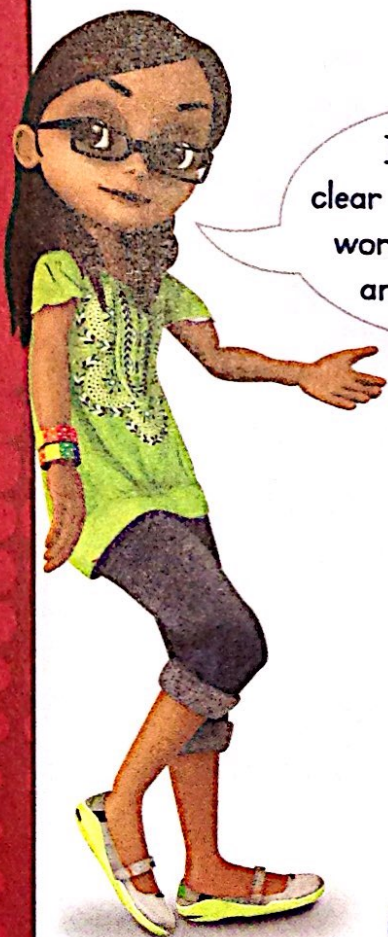
Yes, Molly is correct. Every fraction that has a denominator that is twice as great as its numerator can be written as an equivalent fraction by dividing both the numerator and denominator by the same non-zero number.

$$\frac{5}{10} = \frac{5 \div 5}{10 \div 5} = \frac{1}{2}$$

### Thinking Habits

Be a good thinker! These questions can help you.

- How can I use numbers, objects, drawings, or actions to justify my argument?
- Am I using numbers and symbols correctly?
- Is my explanation clear and complete?
- What questions can I ask to understand other people's thinking?
- Are there mistakes in other people's thinking?
- Can I improve other people's thinking?
- Can I use a counterexample in my argument?





4

## Model with mathematics.

Good math thinkers choose and apply math they know to show and solve problems from everyday life.



I can use what I know about division to solve this problem. I can draw a picture to help.



Jasmine has a strip of wood that is 75 centimeters long. She is going to saw it into 5 equal pieces. How long will each piece of wood be if the strip is cut into 5 equal pieces?

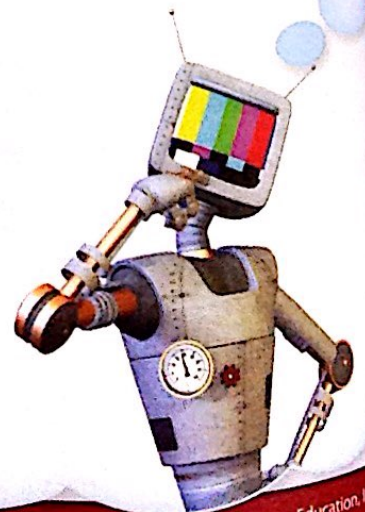


$$75 \div 5 = ?$$

### Thinking Habits

*Be a good thinker! These questions can help you.*

- How can I use math I know to help solve this problem?
- How can I use pictures, objects, or an equation to represent the problem?
- How can I use numbers, words, and symbols to solve the problem?





5

## Use appropriate tools strategically.

Good math thinkers know how to choose the right tools to solve math problems.



I decided to use a protractor because I could measure the angles directly.



Harry said that the angle made at the back point of home plate is an acute angle. Is Harry correct? Justify your argument.

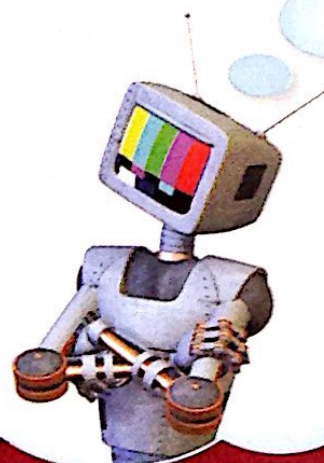


Harry is incorrect. The angle is a right angle because it has a measure of  $90^\circ$ .

### Thinking Habits

*Be a good thinker! These questions can help you.*

- Which tools can I use?
- Why should I use this tool to help me solve the problem?
- Is there a different tool I could use?
- Am I using the tool appropriately?





## Attend to precision.

Good math thinkers are careful about what they write and say, so their ideas about math are clear.



Bill has 125 oranges. He puts 6 oranges into each box. How many boxes does he need?

$$125 \div 6 = 20 \text{ R}5$$

120 oranges will fit into 20 boxes.  
So, Bill needs 21 boxes for 125 oranges.

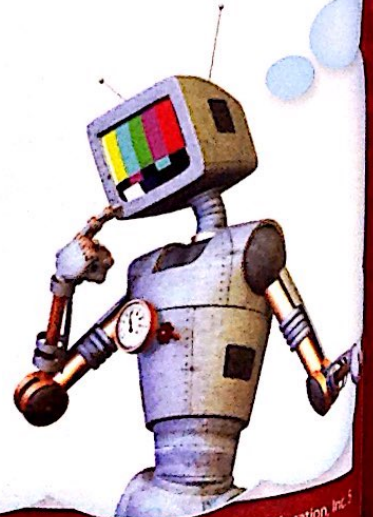
I was precise with my work and the way that I wrote my solution.



### Thinking Habits

Be a good thinker! These questions can help you.

- Am I using numbers, units, and symbols appropriately?
- Am I using the correct definitions?
- Am I calculating accurately?
- Is my answer clear?







## Look for and make use of structure.

Good math thinkers look for patterns or relationships in math to help solve problems.



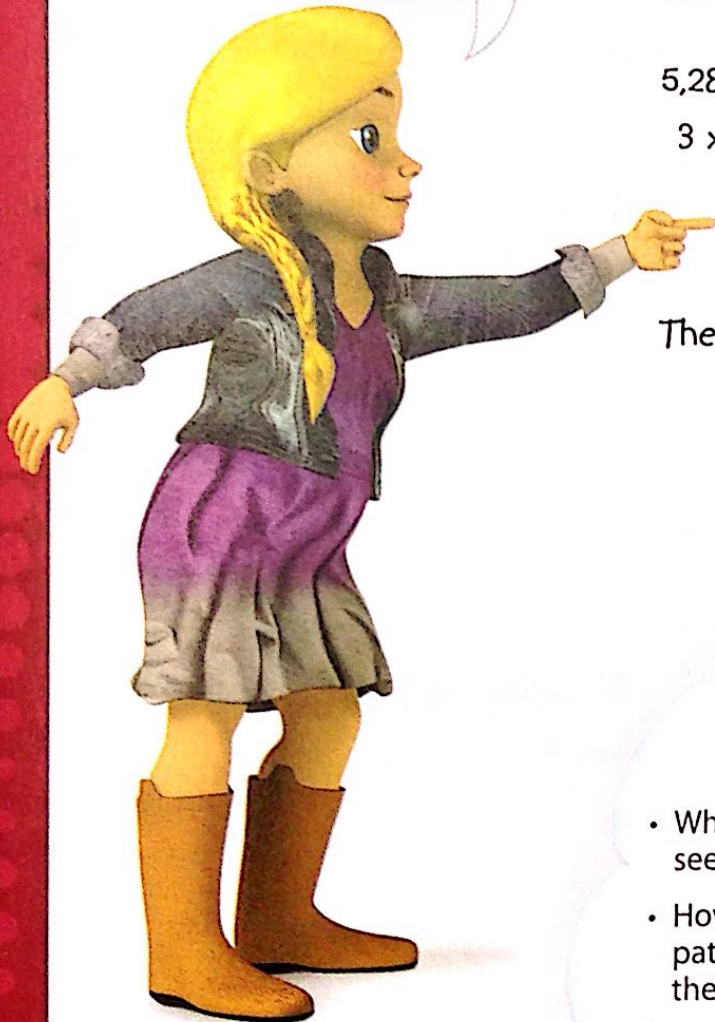
I broke numbers apart to multiply.

There are 5,280 feet in 1 mile.  
How many feet are in 3 miles?

$$5,280 \text{ feet} = 1 \text{ mile}$$

$$\begin{aligned} 3 \times 5,280 &= 3 \times (5,000 + 200 + 80) \\ &= (3 \times 5,000) + (3 \times 200) + (3 \times 80) \\ &= 15,000 + 600 + 240 \\ &= 15,840 \end{aligned}$$

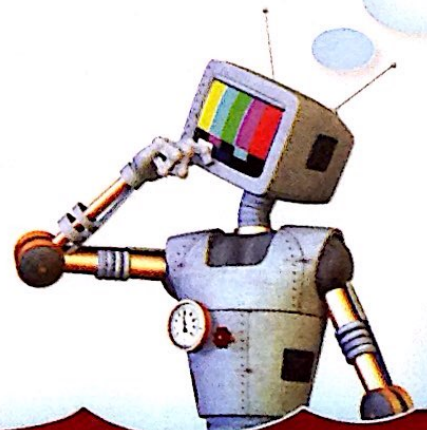
There are 15,840 feet in 3 miles.



### Thinking Habits

*Be a good thinker! These questions can help you.*

- What patterns can I see and describe?
- How can I use the patterns to solve the problem?
- Can I see expressions and objects in different ways?
- What equivalent expressions can I use?

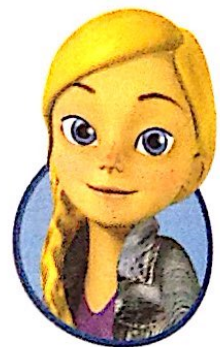




8

## Look for and express regularity in repeated reasoning.

Good math thinkers look for things that repeat, and they make generalizations.



I used reasoning to generalize about calculations.



Use  $<$ ,  $>$ , or  $=$  to compare the expressions without calculating.

$$600 \div 10 \bigcirc 600 \times 10$$

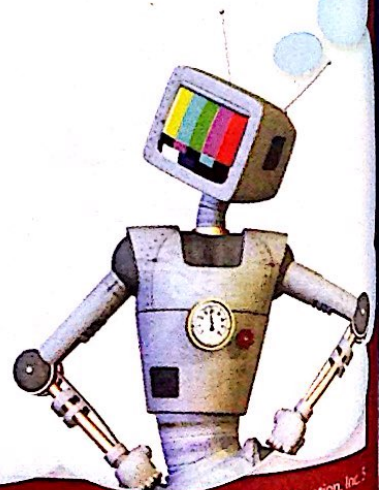
$$600 \div 10 < 600 \times 10$$

Dividing by 10 results in a number less than multiplying by 10.

### Thinking Habits

*Be a good thinker! These questions can help you.*

- Are any calculations repeated?
- Can I generalize from examples?
- What shortcuts do I notice?



# Problem Solving Guide

These questions can help you solve problems.

## Make Sense of the Problem

### Reason Abstractly and Quantitatively

- What do I need to find?
- What given information can I use?
- How are the quantities related?

### Think About Similar Problems

- Have I solved problems like this before?

## Persevere in Solving the Problem

### Model with Math

- How can I use the math I know?
- How can I represent the problem?
- Is there a pattern or structure I can use?

### Use Appropriate Tools Strategically

- What math tools could I use?
- How can I use those tools strategically?

## Check the Answer

### Make Sense of the Answer

- Is my answer reasonable?

### Check for Precision

- Did I check my work?
- Is my answer clear?
- Did I construct a viable argument?
- Did I generalize correctly?



## Some Ways to Represent Problems

- Draw a Picture
- Make a Bar Diagram
- Make a Table or Graph
- Write an Equation

## Some Math Tools

- Objects
- Grid Paper
- Rulers
- Technology
- Paper and Pencil

# Problem Solving Recording Sheet

This sheet helps you organize your work.



Name Carlos

Teaching Tool  
**1**

## Problem Solving Recording Sheet

### Problem:

A store sold 20 sweatshirts. Of these, 8 were red. Twice as many were green as yellow. How many of each color sweatshirt did the store sell?

### MAKE SENSE OF THE PROBLEM

#### Need to Find

How many sweatshirts were sold in each color?

#### Given

A total of 20 sweatshirts.  
8 were red. Twice as many green sweatshirts as yellow.

### PERSEVERE IN SOLVING THE PROBLEM

#### Some Ways to Represent Problems

- Draw a Picture
- Make a Bar Diagram
- Make a Table or Graph
- Write an Equation

#### Some Math Tools

- Objects
- Grid Paper
- Rulers
- Technology
- Paper and Pencil

#### Solution and Answer

$20 - 8 = 12$ , so there are 12 green and yellow sweatshirts. If there are 2 green shirts, there will be 1 yellow shirt.

green	yellow	total
2	1	3
4	2	6
6	3	9
8	4	12

So, there are 8 green sweatshirts and 4 yellow sweatshirts.

### CHECK THE ANSWER

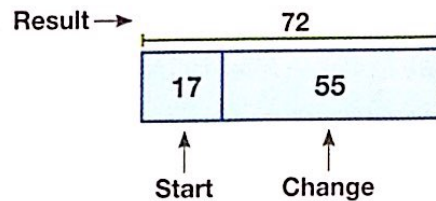
I can add to check my work. 8 red, 8 green, and 4 yellow sweatshirts.  $8 + 8 + 4 = 20$ . There are 20 sweatshirts in all.

# Bar Diagrams

You can draw a **bar diagram** to show how the quantities in a problem are related. Then you can write an equation to solve the problem.

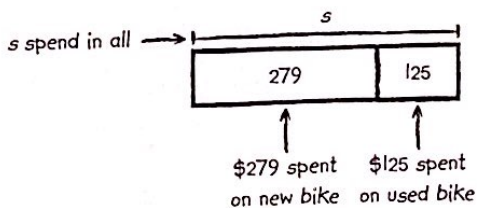
## Add To

Draw this **bar diagram** for situations that involve *adding* to a quantity.



### Result Unknown

Monica bought a new bicycle for \$279. She also bought a used bicycle for \$125. How much did she spend in all?



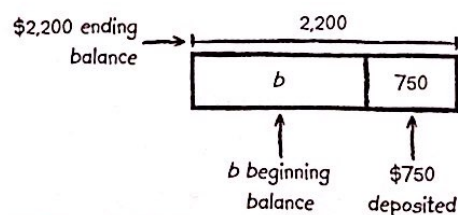
$$279 + 125 = s$$

Monica spent \$404 on the two bikes.

### Start Unknown

Vanessa deposited \$750 in her bank account. After she made the deposit, she had \$2,200 in her account. How much did Vanessa start with in her account?

ONLINE BANKING	
Account	Vanessa
Beginning balance	?
Deposit	\$750.00
Ending balance	\$2,200.00
Transfer	



$$b + 750 = 2,200$$

Vanessa started with \$1,450.

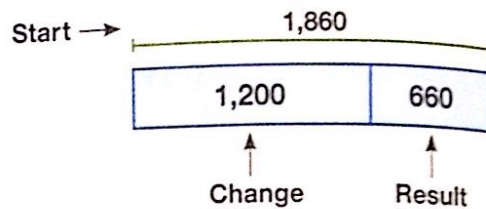
# Bar Diagrams

You can use bar diagrams to make sense of addition and subtraction problems.



## Take From

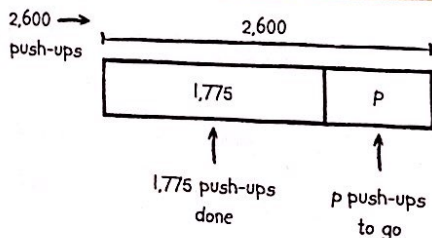
Draw this **bar diagram** for situations that involve *taking* from a quantity.



## Result Unknown

Nicolas has a goal of doing 2,600 push-ups this year. He has done 1,775 push-ups so far. How many more push-ups does he need to do to reach his goal?

Exercise Goals			
Exercise	Goal	Completed	Left to do
push-ups	2,600	1,775	
sit-ups	1,300	900	
chin-ups	520	350	

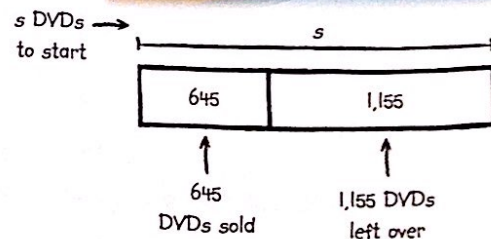


$$2,600 - 1,775 = p$$

Nicolas has 825 more push-ups to do to reach his goal.

## Start Unknown

A store had a collection of DVDs. They sold 645 DVDs during a weekend sale. How many DVDs did the store have before the sale?



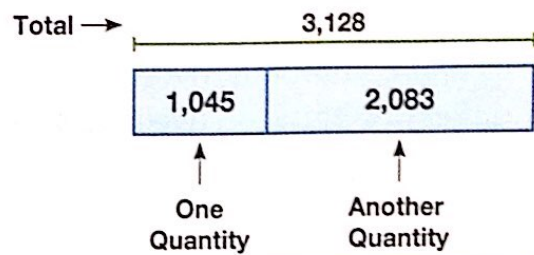
$$s - 645 = 1,155$$

The store had 1,800 DVDs before the sale.

The **bar diagrams** on this page can help you make sense of more addition and subtraction situations.

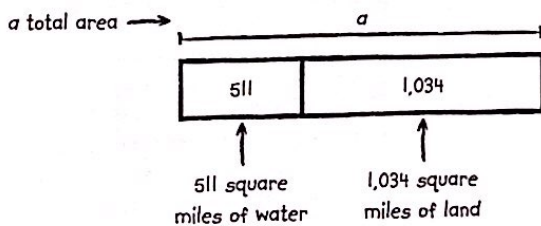
## Put Together/Take Apart

Draw this **bar diagram** for situations that involve *putting together* or *taking apart* quantities.



### Whole Unknown

Rhode Island covers the least space of all states in the U.S. What is the total land and water area of Rhode Island?

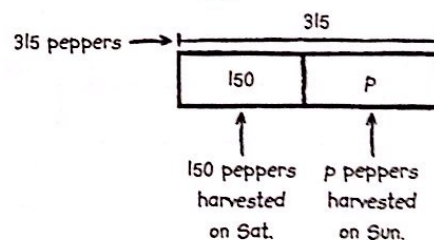


$$511 + 1,034 = a$$

The total land and water area of Rhode Island is 1,545 square miles.

### Part Unknown

A farmer harvested 150 peppers on Saturday. He harvested more peppers on Sunday. He collected a total of 315 peppers over the two days. How many peppers did he harvest on Sunday?



$$150 + p = 315 \text{ or } 315 - 150 = p$$

He harvested 165 peppers on Sunday.

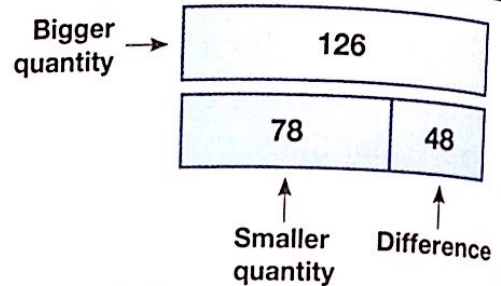
# Bar Diagrams

Pictures help you understand.



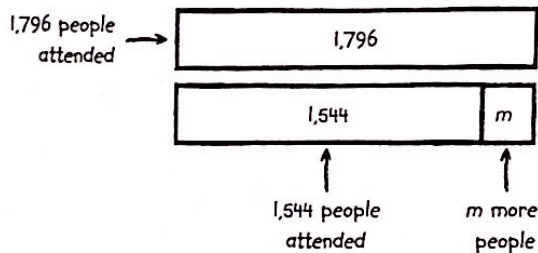
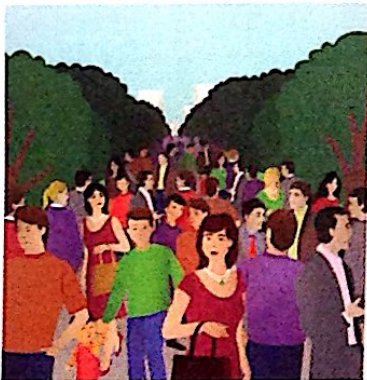
## Compare: Addition and Subtraction

Draw this **bar diagram** for *compare* situations involving the difference between two quantities (how many more or fewer).



### Difference Unknown

Last year, 1,796 people attended the county fair. This year 1,544 people attended. How many more people attended last year than this year?

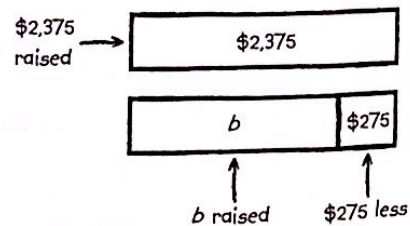
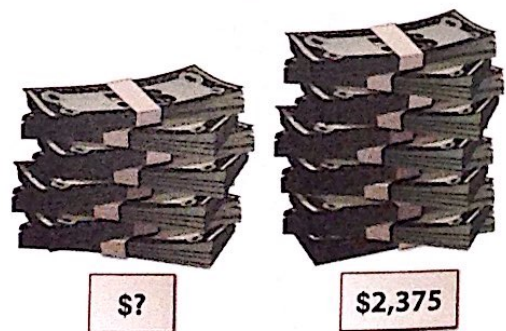


$$1,796 - 1,544 = m$$

Last year, 252 more people attended.

### Smaller Unknown

Ann's school raised \$2,375 for charity. Brian's school raised \$275 less than Ann's school. How much did Brian's school raise?



$$2,375 - b = 275 \text{ or } b + 275 = 2,375$$

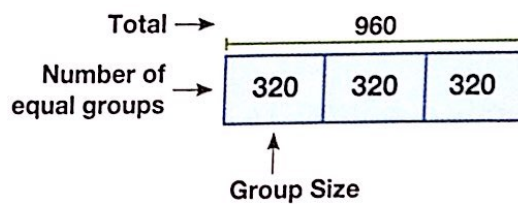
Brian's school raised \$2,100.



The **bar diagrams** on this page can help you solve problems involving multiplication and division.

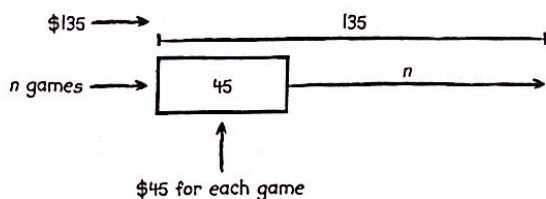
## Equal Groups: Multiplication and Division

Draw this **bar diagram** for situations that involve *equal groups*.



### Number of Groups Unknown

Tom spent \$135 on some new video games. Each game cost the same. How many video games did he buy?

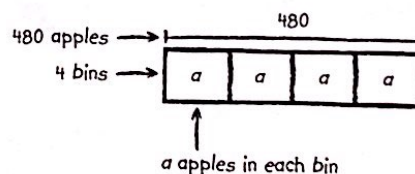
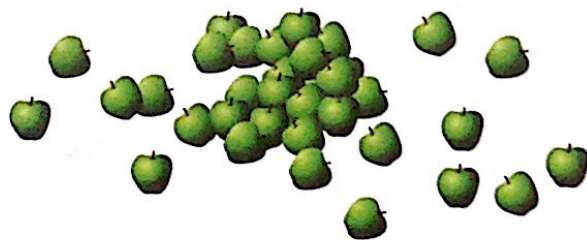


$$135 \div n = 45 \text{ or } n \times 45 = 135$$

Tom bought 3 video games.

### Group Size Unknown

Workers at an orchard harvested 480 apples. They separated the apples evenly into 4 bins. How many apples did they put in each bin?



$$4 \times a = 480 \text{ or } 480 \div 4 = a$$

They put 120 apples in each bin.

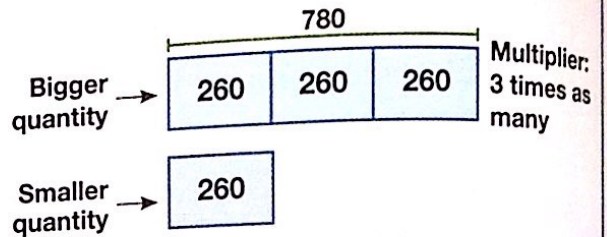
# Bar Diagrams

Bar diagrams can be used to show how quantities that are being compared are related.



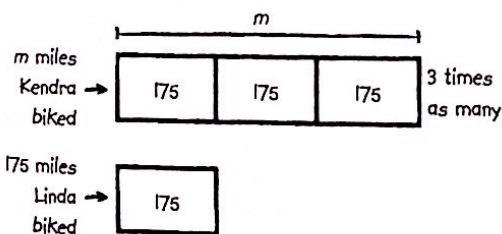
## Compare: Multiplication and Division

Draw this **bar diagram** for *compare* situations involving how many times one quantity is of another quantity.



### Bigger Unknown

Linda biked 175 miles last summer. Kendra biked 3 times as far as Linda. How many miles did Kendra bike?

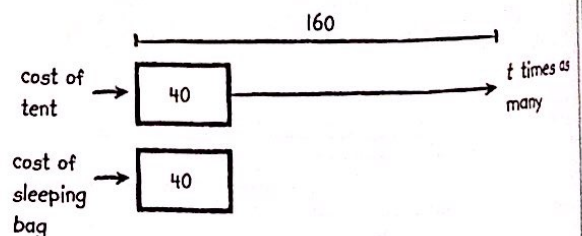


$$3 \times 175 = m$$

Kendra biked 525 miles.

### Multiplier Unknown

Joe buys a new tent and sleeping bag. How many times as much as the sleeping bag does the tent cost?



$$160 \div 40 = t \text{ or } 40 \times t = 160$$

The tent costs 4 times as much as the sleeping bag.